
Submit the solutions in groups of two at the lecture on Tuesday, 2018-06-19

Exercise 1. Let $z : \mathbb{R} \rightarrow \mathbb{C}$ be a Lipschitz chord-arc curve and f a smooth compactly supported function. Show that

$$\lim_{\delta \rightarrow 0} \int \frac{f(y)z'(y)dy}{z(y) - z(x) + i\delta z'(x)} = Cf(x) + \lim_{\epsilon \rightarrow 0} \int_{|x-y|>\epsilon} \frac{f(y)z'(y)dy}{z(y) - z(x)}$$

for every $x \in \mathbb{R}$ such that $z'(x)$ exists.

Hints: reduce to the case $x = z(x) = 0$ if you find this helpful. Split $f = \int_{|x-y|>\epsilon} + \int_{|x-y|<\epsilon}$ and $f(y) = f(x) + (f(y) - f(x))$. Let first $\delta \rightarrow 0$ and then $\epsilon \rightarrow 0$.

Exercise 2. Let T be a CZO associated to a CZ kernel K and $0 < r \leq 1$. Let

$$T_{\sharp}f(x) := \sup_{\epsilon > 0, |x-x'| \leq \epsilon/2} \left| \int_{\mathbb{R}^d \setminus B(x', \epsilon)} K(x', y)f(y)dy \right|.$$

Show that

$$T_{\sharp}f \lesssim Mf + M(|Tf|^r)^{1/r}.$$

Advertisement. The student council of mathematics will organize the math party on 21/06 in N8schicht. The presale will be held on Mon 18/06, Tue 19/06 and Wed 20/06 in the mensa Poppelsdorf. Further information can be found at fsmath.uni-bonn.de